Lecture 17: Morphology (ch 7) & Image Matching (ch 13)

ch. 7 and ch. 13 of *Machine Vision* by Wesley E. Snyder & Hairong Qi

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16-725 (CMU RI) : BioE 2630 (Pitt)

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Mathematical Morphology

- The study of shape…
- Using Set Theory

- Most easily understood for binary images.

Binary Morphology: Basic Idea

1. Make multiple copies of a shape
2. Translate those copies around
3. Combine them with either their:
   - Union, $\cup$, in the case of dilation, $\oplus$
   - Intersection, $\cap$, in the case of erosion, Θ

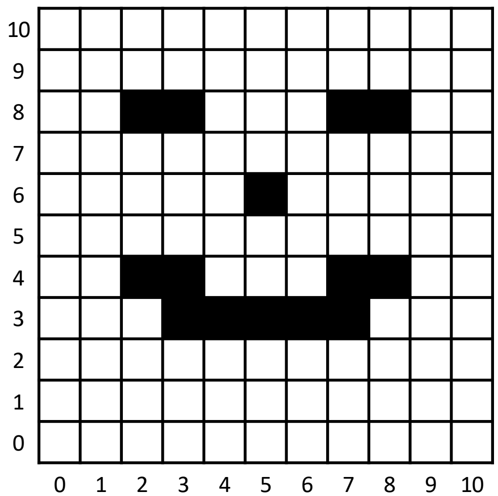
Dilation makes things bigger
Erosion makes things smaller

Binary Morphology: Basic Idea

- Q: How do we designate:
  - The number of copies to make?
  - The translation to apply to each copy?
- A: With a structuring element (s.e.)
  - A (typically) small binary image.
  - We will assume the s.e. always contains the origin.
- For each marked pixel in the s.e.:
  - Make a new copy of the original image
  - Translate that new copy by the coordinates of the current pixel in the s.e.

Dilation Example

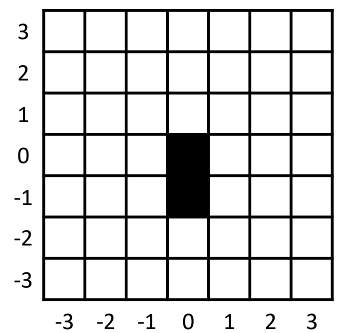
f_A



A = { (2,8),(3,8),(7,8),(8,8),(5,6),(2,4),(3,4),(3,3),(4,3),(5,3),(6,3),(7,3),(7,4),(8,4) }

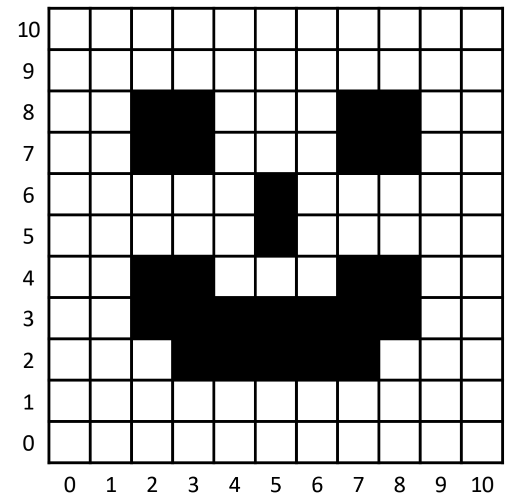
$\oplus$

f_B = s.e.



B = { (0,0),(0,-1) }

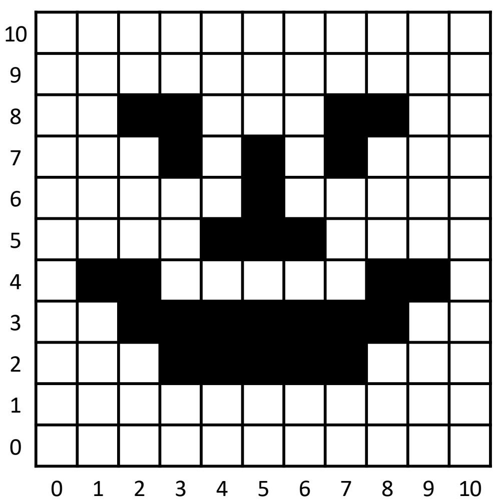
=



Erosion Example

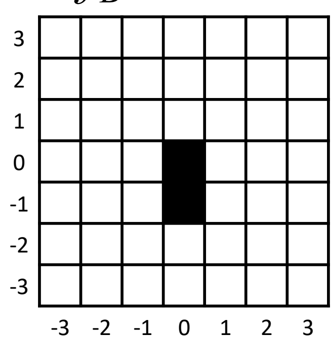
- For erosion, we translate by the *negated* coordinates of the current pixel in the s.e.

f_A



Θ

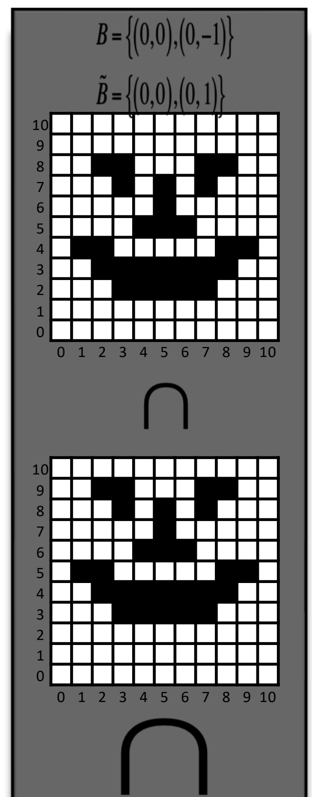
f_B = s.e.



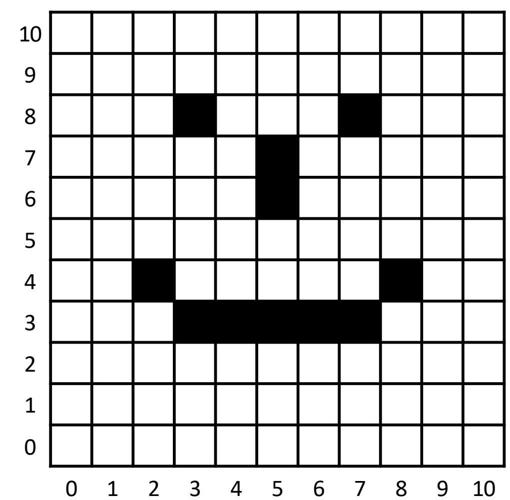
$$B = \{(0,0),(0,-1)\}$$

$$\tilde{B} = \{(0,0),(0,1)\}$$

=

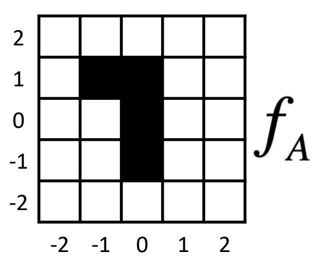


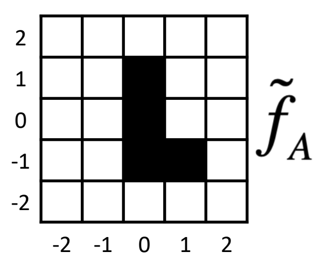
=



Notation

- A (binary) image: f_A
- The set of marked pixels in f_A: A
  - A = { (x_1,y_1), (x_2,y_2), … }
- A translated image or set: $f_{A_{(dx,dy)}}$ or $A_{(\mathrm{dx,dy})}$
- The number of elements in A: #A
- Complement (inverse) of A: A^c
- Reflection (rotation) of A: Ã
  - Ã = { $(-x,-y)$ | (x,y) ∈ A }





Properties

- Dilation:
  - Commutative, Associative, & Distributive
  - Increasing: If $A \subseteq B$ then $A \oplus K \subseteq B \oplus K$
  - Extensive: $A \subseteq A \oplus B$
- Erosion:
  - Anti-extensive ($A \Theta B \subseteq A$), … (see the text)
- Duality:
  - $(A \Theta B)^c = A^c \oplus \widetilde{B}$
  - $(A \oplus B)^c = A^c \Theta \widetilde{B}$
- Not Inverses:
  - $A \neq (A \Theta B) \oplus B$
  - $A \neq (A \oplus B) \Theta B$

This is actually the *opening* of A by B

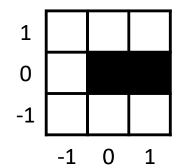
This is actually the *closing* of A by B

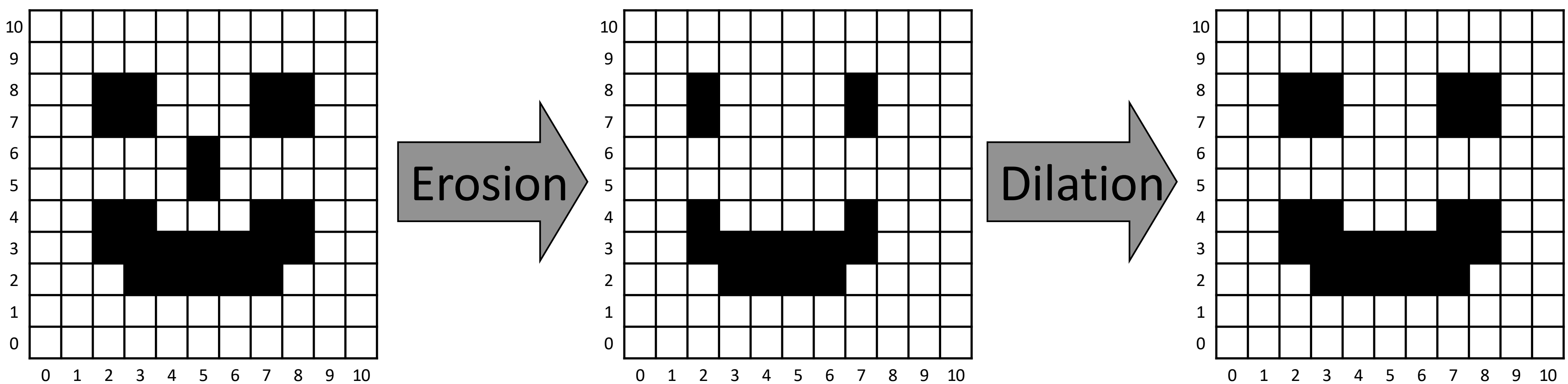
Opening

- $f_A \circ f_B = (f_A \ominus f_B) \oplus f_B$
- Preserves the geometry of objects that are "big enough"
- Erases smaller objects
- Mental Concept:
  - "Pick up" the s.e. and place it in f_A.
  - Never place the s.e. anywhere it covers any pixels in f_A that are not marked.
  - $f_A \circ f_B$ = the set of (marked) pixels in f_A which can be covered by the s.e.

Opening Example

- Use a horizontal s.e. to remove 1-pixel thick vertical structures:



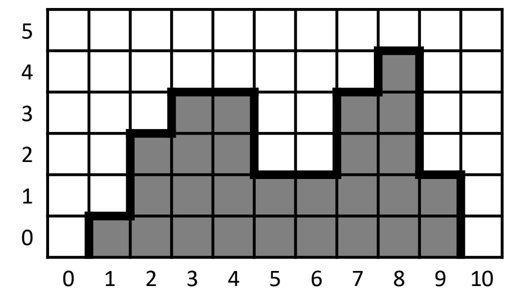


Gray-Scale Morphology

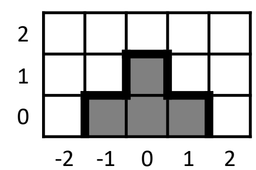
- Morphology operates on sets
- Binary images are just a set of marked pixels
- Gray-scale images contain more information
- How can we apply morphology to this extra intensity information?
- We need to somehow represent intensity as elements of a set

The Umbra

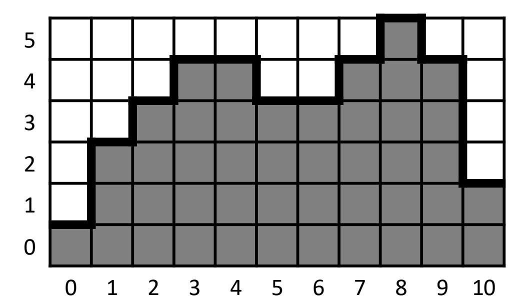
- Gray-scale morphology operates on the umbra of an image.
- Imagine a 2D image as a pixilated surface in 3D
- We can also “pixilate” the height of that surface
- The 2D image is now a 3D surface made of 3D cells



The umbra of a 1D image



The umbra of a 1D s.e.



Dilation

The Distance Transform (DT)

- Records at each pixel the distance from that pixel to the nearest boundary (or to some other feature).
- Used by other algorithms
- The DT is a solution of the Diff. Eq.:

  $|| \nabla DT(x) || = 1$,

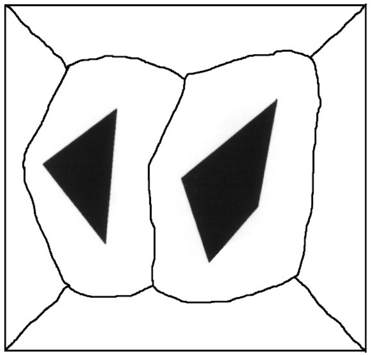
  $DT(x) = 0$ on boundary
- Can compute using erosion
  - $DT(x)$ = iteration when x disappears
  - Details in the book

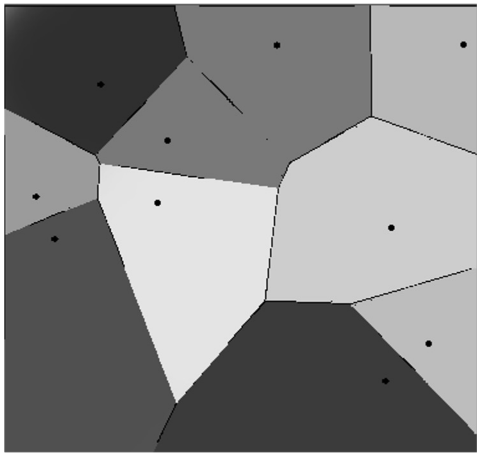
	1					1	1	
	1					1	1	
	1	1	1		1	**2**	1	
	1	**2**	**2**	1	**2**	**2**	1	
		1	**2**	**2**	**3**	**2**	1	
		1	**2**	**3**	**2**	**2**	1	
		1	**2**	**2**	1	**2**	1	
		1	**2**	1		1	1	
			1			1		
			1					

DT of a region's *interior*

Voronoi Diagram

- **Divides space**
- Related to DT
- Q: To which of a set of regions (or points) is this point the closest?
- Voronoi Diagram's boundaries = points that are equi-distant from multiple regions
- Voronoi Domain of a region = the "cell" of the Voronoi Diagram that contains the region
- Details in the text





The voronoi diagram of a set of 10 points is public domain from:
http://en.wikipedia.org/wiki/File:2Ddim-L2norm-10site.png

Imaging Matching (ch. 13)

- Matching iconic images
- Matching graph-theoretic representations

- Most important:
  - Eigenimages
  - Springs & Templates

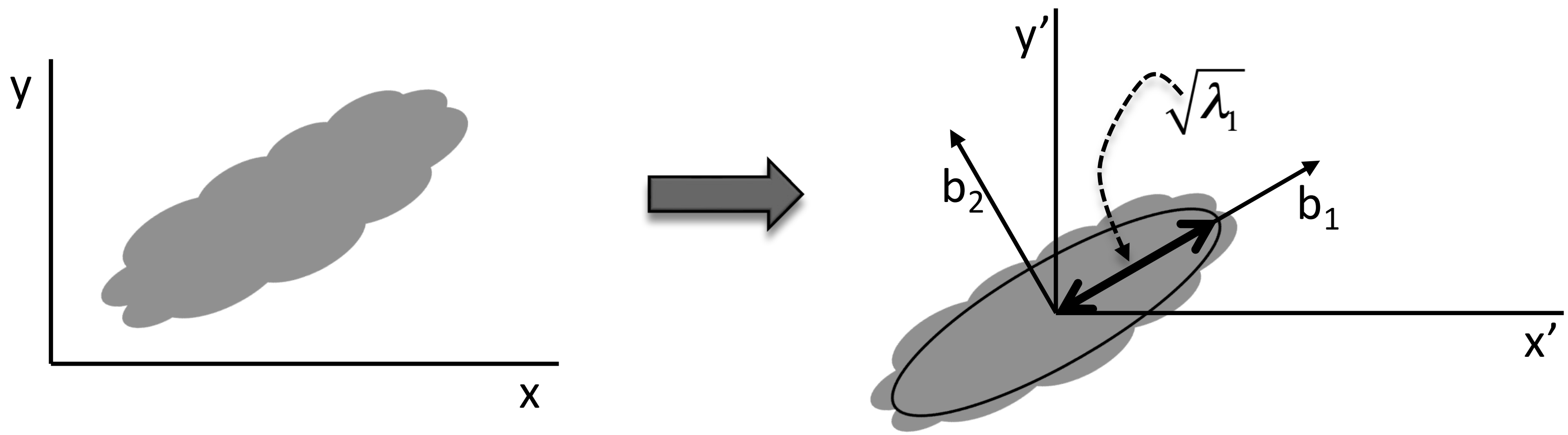
Template Matching

- Template ≈ a relatively small reference image for some feature we expect to see in our input image.
- Typical usage: Move the template around the input image, looking for where it "matches" the best (has the highest correlation).
- Rotation & scale can be problematic
  - Often require multiple passes if they can't be ruled out a-priori
- How "big" do we make each template?
  - Do we represent small, simple features
  - Or medium-size, more complex structures?

Eigenimages

- Goal: Identify an image by comparing it to a database of other images
- Problem: Pixel-by-pixel comparisons are two expensive to run across a large database
- Solution: Use PCA

PCA (K-L Expansion)



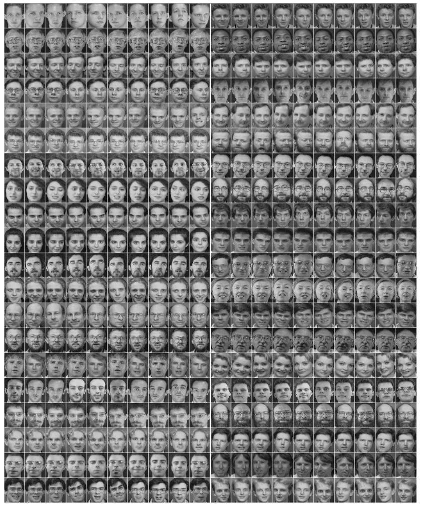
- **Big Picture:** Fitting a hyper-ellipsoid & then (typically) reducing dimensionality by flattening the shortest axes
- Same as fitting an (N+1)-dimensional multivariate Gaussian, and then taking the level set corresponding to one standard deviation
- Mathematically, PCA reduces the dimensionality of data by mapping it to the first n eigenvectors (*principal components*) of the data's covariance matrix
- The first principal component is the eigenvector with the largest eigenvalue and corresponds to the longest axis of the ellipsoid
- The variance along an eigenvector is exactly the eigenvector's eigenvalue
- This is VERY important and VERY useful. Any questions?

Eigenimages: Procedure

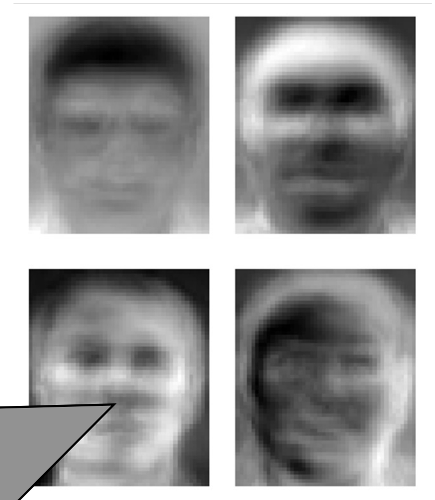
- Run PCA on the training images
  - See the text for efficiency details
- Store in the database:
  - The set of dominant Eigenvectors
    - = the principle components
    - = the Eigenimages
  - For each image, store its coefficients when projected onto the Eigenimages
- Match a new image:
  - Project it onto the basis of the Eigenimages
  - Compare the resulting coefficients to those stored in the database.

Eigenimages Example

Training Images



Eigenimages / Eigenfaces



PCA

Project Onto

New Images:

Which training image(s) does each face most resemble?

The face database and the derived Eigenface examples are all from AT&T Laboratories Cambridge:

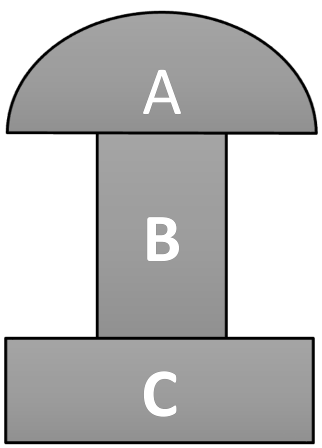
http://www.cl.cam.ac.uk/research/dtg/attarchive/facedatabase.html & http://en.wikipedia.org/wiki/File:Eigenfaces.png

Matching Simple Features

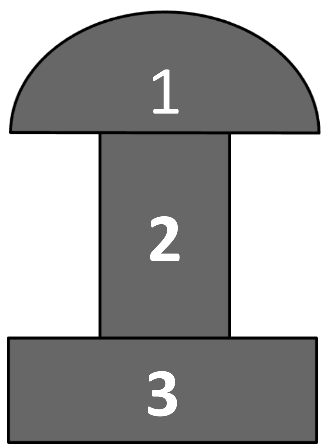
- Classification based on features
  - Ex: mean intensity, area, aspect ratio
- Idea:
  - Combine a set of shape features into a single feature vector
  - Build a statistical model of this feature vector between and across object classes in a sequence of training shapes
  - Classification of a new shape = the object class from which the new shape's feature vector most likely came.

Graph Matching: Association Graphs

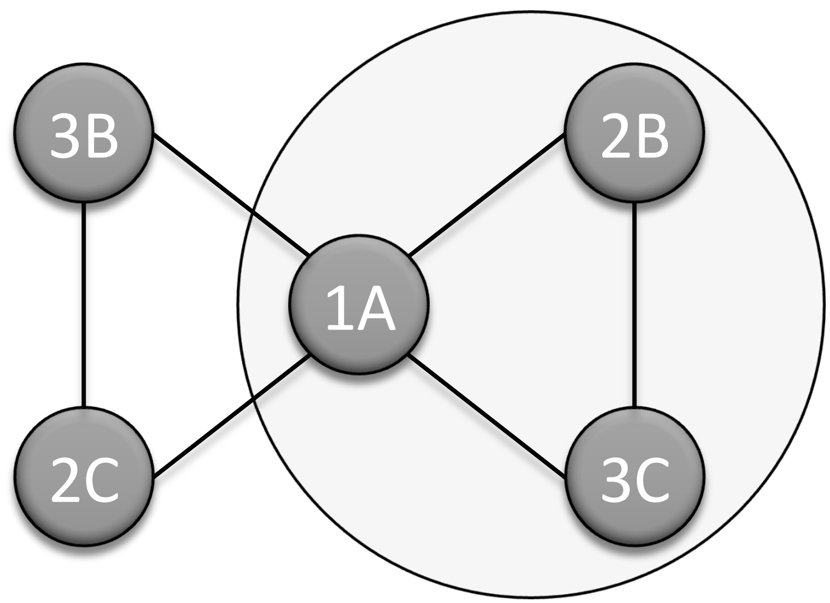
- Match nodes of model to segmented patches in image
- Maximal cliques represent the most likely correspondences
  - Clique = a totally connected subgraph
- Problems: Over/under segmentation, how to develop appropriate rules, often > 1 maximal clique



Image



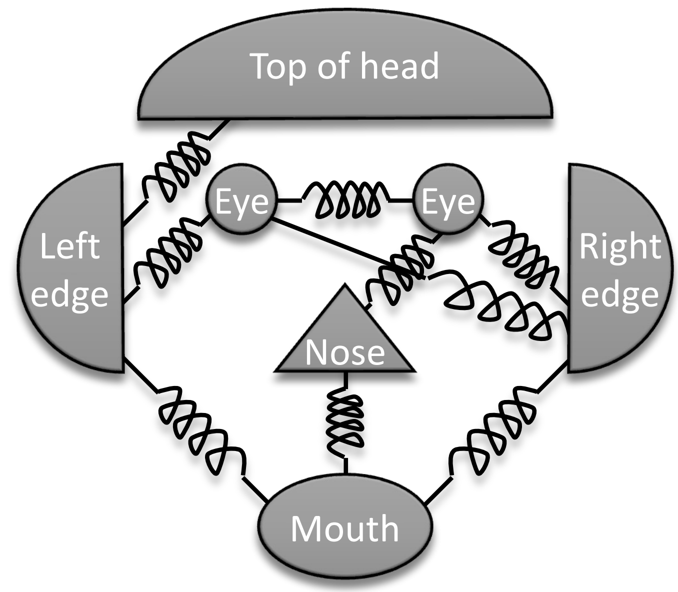
Model



One of two maximal cliques

Graph Matching: Springs & Templates

- Idea: When matching simple templates, we usually expect a certain arrangement between them.
- So, arrange templates using a graph structure.
- The springs are allowed to deform, but only “so” much.



Fischler and Elschlager’s “Pictorial Structures” spring & template model for image matching from the early 1970s

Graph Matching: Springs & Templates

- A match is based on minimizing a total cost.
- Problem: Making sure missing a point doesn't improve the score.

$$\text{Cost} =$$

$$\sum_{d \in templates} \text{TemplateCost}\left(d, F(d)\right)$$

$$+$$

$$\sum_{d,e \in ref \times ref} \text{SpringCost}\left(F(d), F(e)\right)$$

$$+$$

$$\sum_{c \in \left(R_{missing} \times R_{missing}\right)} \text{MissingCost}\left(c\right)$$